

Trans. vol. clv. p. 653); and as I there, in the case of three variables, introduced a set of three arbitrary constants in order to comprise a group of expressions in a single formula, so here, in the case of four variables, I introduce with the same view two sets of four arbitrary constants. If these constants be represented by $\alpha, \beta, \gamma, \delta, \alpha', \beta', \gamma', \delta'$, I consider the conic of five-pointic contact of a section of the surface made by the plane $\varpi - k\varpi' = 0$, where $\varpi = \alpha x + \beta y + \gamma z + \delta t$, and $\varpi' = \alpha' x + \beta' y + \gamma' z + \delta' t$, and k is indeterminate; and then proceed to determine k , and thereby the azimuth of the plane about the line $\varpi = 0, \varpi' = 0$, so that the contact may be six-pointic. The formulæ thence arising turn out to be strictly analogous to those belonging to the case of three variables, except that the arbitrary quantities cannot in general be divided out from the final expression. In fact, it is the presence of these quantities which enables us to determine the position of the plane of section, and the equation whereby this is effected proves to be of the degree 10 in $\varpi : \varpi' = k$, and besides this of the degree $12n - 27$ in the coordinates x, y, z, t (n being the degree of the surface), giving rise to the theorem above stated.

Beyond the question of the principal tangents, it has been shown by Clebsch and Salmon that on every surface U a curve may be drawn, at every point of which one of the principal tangents will have a four-pointic contact. And if n be the degree of U , that of the surface S intersecting U in the curve in question will be $11n - 24$. Further, it has been shown that at a finite number of points the contact will be five-pointic. The number of these points has not yet been completely determined; but Clebsch has shown (Crelle, vol. lviii. p. 93) that it does not exceed $n(11n - 24) (14n - 30)$. Similarly it appears that on every surface a curve may be drawn, at every point of which one of the principal tangent conics has a seven-pointic contact, and that at a finite number of points the contact will become eight-pointic. But into the discussion of these latter problems I do not propose to enter in the present communication.

March 17, 1870.

Capt. RICHARDS, R.N., Vice-President, in the Chair.

The following communications were read:—

- I. "On the Law which regulates the Relative Magnitude of the Areas of the four Orifices of the Heart." By HERBERT DAVIES, M.D., F.R.C.P., Senior Physician to the London Hospital, and formerly Fellow of Queens' College, Cambridge. Communicated by W. H. FLOWER, Hunterian Professor of Comparative Anatomy. Received January 27, 1870.

I propose in this communication to inquire whether any law can be discovered which determines the relative magnitude of the areas of the

tricuspid, pulmonic, mitral, and aortic orifices—the four principal openings in the heart.

Although to ordinary observation these orifices appear to exhibit no mutual relationship of size, there can be no doubt that an instrument so accurate in the adaptation of its valvular apparatus, and so exact in the working of its different parts, must reveal on close examination the existence of laws which not only determine the force required to be impressed upon the blood traversing its chambers, but also the relative sizes of these apertures to one another.

The facts and inferences which I shall adduce on this subject will tend, I believe, to throw some light upon the mechanism of the heart in its healthy state, and will explain also some points of practical interest in the pathology of that organ.

To M. Bizot in France, and Drs. Peacock and Reid in this country, are we mainly indebted for the most careful and trustworthy measurements of the circumferences of the orifices. Their measurements have been recorded to the minuteness of the thousandth part of an inch, and yet it will, I believe, be readily admitted that the results in the form given by these distinguished observers help us but little in obtaining any definite idea of the mutual relationship of the areas of the orifices, and are destitute of any practical value in our study of the mechanism of the heart itself.

Had the observations been pushed further, or, rather, had the figures been worked out into some distinct and definite shape, these observers could not have failed to discover an interesting and important law presiding over the areas of these orifices, and they would thus have been enabled to utilize a multitude of measurements which had been obtained by considerable labour and patient research.

Taking the measurements given by Dr. Peacock in the Croonian Lectures for 1865, we find the mean circumferences of the four orifices, expressed in English inches, to be as follows:—

	Males.	Females.
Tricuspid	4·74	4·562
Pulmonic	3·552	3·493
Mitral.....	4	3·996
Aortic.....	3·14	3·019

I will now place these valuable facts into another shape by calculating from these measurements of *circumference* the *areas* of the respective openings.

The circumference of a circular opening being known, its radius is determined from the formula

$$\text{circumference} = 2\pi r,$$

where $\pi = 3\cdot1415$; and the radius being thus determined, the *area* of the opening is calculated from the formula

$$\text{area} = \pi r^2.$$

The mean areas of the four orifices thus obtained are found to be as follows:—

	Males.	Females.
Tricuspid.....	1·78 sq. in.	1·6 sq. in.
Pulmonic.....	1	·97
Mitral	1·27	1·27
Aortic	·78	·67

Or, for facility of recollection, we may consider the respective mean areas in the male to be:—

Tricuspid.....	1 $\frac{3}{4}$ sq. in.
Pulmonic.....	1
Mitral	1 $\frac{1}{4}$
Aortic	$\frac{3}{4}$

whence it is obvious that the apertures differ very considerably in area from each other, the tricuspid having the largest area, its orifice being more than double the size of the aortic opening.

Irregular, however, as these areas may appear to be in magnitude with respect to each other, we shall find, on pushing our observations further, that there is a distinct and constant law presiding over them, and this law is discovered when we compare the *ratios* of the areas of corresponding orifices. Thus,

$$\frac{\text{Area of tricuspid}}{\text{Area of mitral}} = \frac{1\cdot78}{1\cdot27} = 1\cdot4, \text{ nearly ;}$$

$$\frac{\text{Area of pulmonic}}{\text{Area of aortic}} = \frac{1}{\cdot78} = 1\cdot3, \text{ nearly ;}$$

or, in other words, the area of the tricuspid appears from these calculations to bear nearly the same relation to the area of the mitral which the area of the pulmonic does to that of the aortic orifice, *i. e.* were the tricuspid, for example, twice the size of the mitral orifice in area, the pulmonic would be twice the size of the aortic orifice in area, the two ratios differing from each other only by one-tenth.

Again, if we adopt Dr. Reid's measurements of the circumference of the healthy cardiac orifices (these measurements being given in Dr. Peacock's work), we shall find this law to be more conclusively proved.

According to Dr. Reid the measurements of the circumferences are as follows:—

	Male.	Female.
Tricuspid	5·3 in.	4·9 in.
Pulmonic	3·7	3·5
Mitral	4·6	4·2
Aortic	3·2	3

from which data we find the areas to be:—

Tricuspid	2.24 sq. in.	1.9 sq. in.
Pulmonic	1.01	1
Mitral.....	1.7	1.4
Aortic.....	.8	.71

And if we make the same comparison of areas as we did in Dr. Peacock's measurements, we find:—

Males.

$$\frac{\text{Area of tricuspid}}{\text{Area of mitral}} = \frac{2.24}{1.7} = 1.31\frac{1}{2}$$

$$\frac{\text{Area of pulmonic}}{\text{Area of aortic}} = \frac{1.01}{.8} = 1.26$$

$$\text{Difference of the ratios} = .05$$

Females.

$$\frac{\text{Area of tricuspid}}{\text{Area of mitral}} = \frac{1.9}{1.4} = 1.36$$

$$\frac{\text{Area of pulmonic}}{\text{Area of aortic}} = \frac{1}{.71} = 1.40$$

$$\text{Difference of the ratios} = .04$$

It is well known that no measurements can be taken of such orifices as those of the heart without liability to error; but no one can observe the close identity of the respective ratios without concluding that the ratios are really identical, and that the small differences in the calculated results depend entirely upon the impossibility of obtaining absolutely correct measurements of the boundaries of such openings. It is clear, therefore, that in whatever proportion the tricuspid is larger than the mitral, in exactly the same proportion is the pulmonic larger in area than the aortic orifice. This rule applies, of course, to the human heart only in its healthy state; but I shall show that its application is of practical value when we consider the organ in its diseased state.

I shall now proceed to prove that the law which I have deduced from independent observations made in the healthy human heart is of far wider application, for I have found by my own measurements that a comparison of the areas of the same orifices in animals reveals the same result.

The following are the facts at which I have arrived by careful and repeated measurements of the cardiac apertures in different animals.

The measurements are individual, and not mean, and therefore less liable to error.

Horse.

	Circumference.	Area.
Tricuspid	9.25 in.	6.8 sq. in.
Pulmonic	6.5	3.6
Mitral	8.2	5.3
Aortic	5.9	2.8

$$\frac{\text{Tricuspid}}{\text{Mitral}} = \frac{6.8}{5.3} = 1.283$$

$$\frac{\text{Pulmonic}}{\text{Aortic}} = \frac{3.6}{2.8} = 1.285$$

$$\text{Difference of the ratios} = .002$$

Donkey.

Tricuspid	6.2 in.	3.06 sq. in.
Pulmonic	4.1	1.34
Mitral	5.5	2.40
Aortic	3.7	1.09

$$\frac{\text{Tricuspid}}{\text{Mitral}} = \frac{3.06}{2.40} = 1.27$$

$$\frac{\text{Pulmonic}}{\text{Aortic}} = \frac{1.34}{1.09} = 1.23$$

$$\text{Difference of the ratios} = .04$$

Ox.

Tricuspid	7.5 in.	4.48 sq. in.
Pulmonic	4.8	1.83
Mitral	6.6	3.47
Aortic	4.2	1.40

$$\frac{\text{Tricuspid}}{\text{Mitral}} = \frac{4.48}{3.47} = 1.29$$

$$\frac{\text{Pulmonic}}{\text{Aortic}} = \frac{1.83}{1.40} = 1.30$$

$$\text{Difference of the ratios} = .01$$

Calf.

Tricuspid	5 in.	2 sq. in.
Pulmonic	3.2	.81
Mitral	4.3	1.47
Aortic	2.7	.58

$$\frac{\text{Tricuspid}}{\text{Mitral}} = \frac{2}{1.47} = 1.36$$

$$\frac{\text{Pulmonic}}{\text{Aortic}} = \frac{.81}{.58} = 1.40$$

$$\text{Difference of the ratios} = .04$$

Sheep.

	Circumference.	Area.
Tricuspid	3.7 in.	1.09 sq. in.
Pulmonic	2.5	.49
Mitral	3.2	.81
Aortic	2.1	.35

$$\frac{\text{Tricuspid}}{\text{Mitral}} = \frac{1.09}{.81} = 1.34$$

$$\frac{\text{Pulmonic}}{\text{Aortic}} = \frac{.49}{.35} = 1.40$$

$$\text{Difference of the ratios} = .06$$

Sheep.

Tricuspid	4.25 in.	1.435 sq. in.
Pulmonic	2.70	.580
Mitral	3.70	1.090
Aortic	2.30	.420

$$\frac{\text{Tricuspid}}{\text{Mitral}} = \frac{1.435}{1.090} = 1.316$$

$$\frac{\text{Pulmonic}}{\text{Aortic}} = \frac{.58}{.42} = 1.380$$

$$\text{Difference of the ratios} = .064$$

Pig.

Tricuspid.....	3.95 in.	1.24 sq. in.
Pulmonic.....	2.55	.51
Mitral	3.50	.97
Aortic	2.25	.40

$$\frac{\text{Tricuspid}}{\text{Mitral}} = \frac{1.24}{.97} = 1.278$$

$$\frac{\text{Pulmonic}}{\text{Aortic}} = \frac{.51}{.40} = 1.275$$

$$\text{Difference of the ratios} = .003$$

Pig.

	Circumference.	Area.
Tricuspid	3·6 in.	1·03 sq. in.
Pulmonic	2·5	·49
Mitral	3·2	·81
Aortic	2·1	·38

$$\frac{\text{Tricuspid}}{\text{Mitral}} = \frac{1·03}{·81} = 1·27$$

$$\frac{\text{Pulmonic}}{\text{Aortic}} = \frac{·49}{·38} = 1·29$$

$$\text{Difference of the ratios} = \overline{·02}$$

Dog.

Tricuspid.....	3·65 in.	1·07 sq. in.
Pulmonic.....	1·9	·287
Mitral.....	3·15	·79
Aortic.....	1·6	·204

$$\frac{\text{Tricuspid}}{\text{Mitral}} = \frac{1·07}{·79} = 1·36$$

$$\frac{\text{Pulmonic}}{\text{Aortic}} = \frac{·287}{·204} = 1·40$$

$$\text{Difference of the ratios} = \overline{·04}$$

Dog.

Tricuspid.....	2·9 in.	·69 sq. in.
Pulmonic.....	1·6	·204
Mitral.....	2·5	·49
Aortic.....	1·4	·156

$$\frac{\text{Tricuspid}}{\text{Mitral}} = \frac{·69}{·49} = 1·40$$

$$\frac{\text{Pulmonic}}{\text{Aortic}} = \frac{·204}{·156} = 1·31$$

$$\text{Difference of the ratios} = \overline{·09}$$

From these facts we may fairly conclude that in the healthy human heart, and most probably in the hearts of most animals, the areas of the four apertures bear an exact mathematical relationship to each other, and consequently that if the areas of any three of the openings be known, the area of the fourth orifice can be correctly calculated.

I need scarcely dwell upon the importance of a knowledge of this law in estimating the amount of contraction or dilatation of orifice which a morbid specimen may present. I will, however, now show from my own measure-

ments how this law was applied, and how closely the observed and calculated results agreed in the case of a strong healthy man who died in the London Hospital from the effects of a fractured spine. The heart was perfectly healthy. I carefully measured the pulmonic, mitral, and aortic orifices, calculated the area of the tricuspid, and then measured its circumference. Having worked out its area, I was able to observe what difference existed between the result of actual measurement and the result derived from "the law of the orifices" which I had discovered.

Human Heart.

	Circumference.	Area.
Pulmonic	3·55 in.	1·003 sq. in.
Mitral	4·20	1·405
Aortic	3·10	·765

Now, by the law of the orifices,

$$\frac{\text{Area of tricuspid}}{\text{Area of mitral}} = \frac{\text{Area of pulmonic}}{\text{Area of aortic}} ;$$

$$\therefore \text{area of tricuspid} = 1·405 \times \frac{1·003}{·765}$$

$$= 1·972.$$

By measurement,—

Circumference.	Area.
Tricuspid = 5·1 in.	2·070 sq. in.

$$\begin{aligned} \text{Area of tricuspid by measurement} &= 2·070 \text{ sq. in.} \\ \text{Area of tricuspid by calculation} &= 1·972 \\ \hline \text{Difference between calculated and} & \\ \text{observed results} &= ·098 \end{aligned}$$

The calculated and observed results differ so little from each other, that this case evidently strongly corroborates the correctness of the law which I believe regulates the relative magnitude of the areas of the four cardiac openings.

If, moreover, we scrutinize the measurements, we shall observe an equally important fact, that the ratio of the areas of any two corresponding orifices is almost constant in the same, and, I may almost add, in all animals, man included.

Thus the area of the tricuspid is nearly 1·3 times the area of the mitral orifice, and the area of the pulmonic of course bears the same proportion to that of the aortic opening. By measuring, therefore, the two orifices of the right (supposed healthy), we are enabled by this law to deduce approximately the magnitude of the areas of those of the left heart, and *vice versa*. One healthy orifice being known, the area of the corresponding opening in the other side of the heart can be approximately calculated; and should the latter be diseased, its deviation from the normal area can be

determined, and the amount of abnormal contraction or dilatation fairly estimated.

To illustrate the value of this approximative law, I will exemplify, in a case of mitral constriction detailed by Dr. Walshe (*Diseases of the Heart* p. 373), the mode in which the amount of constriction may be calculated.

Mitral Constriction.

	Circumference.	Area.
Tricuspid	$4\frac{7}{8}=4.875$ in.	1.9 sq. in.
Pulmonic	$3\frac{1}{8}=3.125$	.77
Mitral	$1\frac{7}{8}=1.875$	.28
Aortic	$2\frac{3}{8}=2.375$	.45

$$\frac{\text{Tricuspid}}{\text{Mitral}} = \frac{1.9}{.28} = 7, \text{ nearly.}$$

$$\frac{\text{Pulmonic}}{\text{Aortic}} = \frac{.77}{.45} = 1.7, \text{ nearly.}$$

Hence the tricuspid (by reason of the extreme narrowing of the mitral opening) is seven times larger in area than the latter orifice, in place of being only 1.3 to 1.4 times larger in area. If we suppose the tricuspid to be nearly normal, then as

$$\begin{aligned} \frac{\text{Area of tricuspid}}{\text{Area of mitral}} &= 1.3, \text{ nearly;} \\ \therefore \text{area of mitral (healthy)} &= \frac{\text{area of tricuspid}}{1.3} \\ &= \frac{1.9}{1.3} = 1.45 \text{ sq. in.} \end{aligned}$$

Hence the amount of the contraction of

$$\begin{aligned} \text{the mitral orifice} &= 1.45, \text{ the normal size} \\ &- .28, \text{ its actual size.} \\ &= 1.17 \text{ sq. in.} \end{aligned}$$

“The diseased aperture just admitted the end of the index figure; its edge was rugose, and the valve was funnel-shaped towards the ventricle. The left auricle was much hypertrophied, its walls in some parts being $\frac{1}{4}$ inch in thickness, and its endocardium creaked on being touched.” The pulmonic is evidently large in proportion to the aortic opening (the ratio being 1.7 instead of 1.3 to 1.4); and there was no doubt considerable hypertrophy and dilatation of the right ventricle. The increase in the area of the pulmonic aperture was the direct result of this condition of the right side of the heart. The tricuspid was also probably somewhat dilated, as the “valves looked insufficient to fill the widened orifice,” and the jugular veins appeared during life to be swollen and pulsatory; but the absolute size of the tricuspid shows that the dilatation was not excessive. The area of the aortic opening appears to be below the mean amount. Was

this the result of the small supply of blood which the left ventricle received and impelled into the general system? In any case a knowledge of the existence of this law enables us to read the measurements of the orifices and their respective ratios with increased interest.

It would be interesting to pursue the application of this law in the study of the various forms of valvular disease. I purpose, however, to return to this subject at the end of this paper, and shall seek now to trace out the reasons why the four orifices present such differences in the magnitude of their areas.

And as the foundations of our arguments we must admit the truth of the two following propositions:—

1st. That the ventricles and auricles act exactly synchronously respectively; and 2ndly, that equal volumes of blood pass in exactly equal and the same times respectively through any two corresponding orifices of the healthy heart.

1. "If we examine," says M. Marey "the lines traced by the right and left ventricles, we find a most perfect synchronism in the respective commencements and terminations of their contraction."

"The examination also of a heart exposed during life confirms the deduction; for if we grasp the auricles or the ventricles, we cannot detect the smallest interval between the contractions of parallel cavities."

Again. Stethoscopic examination of the heart demonstrates the existence of only one first sound and of only one second sound, although the causes producing each of those sounds are twofold, inasmuch as they really reside in two (right and left hearts), placed in close and intimate apposition to one another. Under rare circumstances the sound which results from the closure of the semilunar valves has been found reduplicated; but although such an event may occur from the non-synchronous fall of the valves, it is clear that an unimpeded and uninterrupted circulation could not be maintained unless the two sides of the heart, or really the two hearts, contracted and dilated exactly synchronously. Whether the organ acts violently or feebly, with regularity or intermittently, the auscultator detects but two sounds; and even when its valves are diseased, its orifices irremediably altered in diameter, and its muscular walls hypertrophied or atrophied, we find the same law of synchronism presiding over the heart and its sounds, normal or abnormal.

Lastly. An examination by dissection of the fibres which compose the walls of the ventricles, conclusively proves that these chambers must inevitably act exactly synchronously. In Dr. Pettigrew's masterly account of the arrangement of the muscular fibres in the ventricles of vertebrate animals, we find the following remarks made upon this point:—"The fibres of the right and left ventricles anteriorly and septally are to a certain extent independent of each other; whereas posteriorly many of them are common to both ventricles; i. e. *the fibres pass from the one ventricle to the other.*" The drawings 49 and 50 in the memoir clearly prove how "the common

fibres pass from the left to the right ventricle and dip in or bend at the track of the anterior coronary artery to become continuous with fibres having a similar direction in the septum”*.

2. In the next place, it must be admitted that equal volumes of blood pass in exactly equal and the same times through any two corresponding orifices of the heart; for if, for example, we could suppose the quantity thrown out through the pulmonic orifice into the lungs to be *persistently* greater than the amount thrown out in *the same time* through the aortic opening into the general circulation, it would inevitably follow that overwhelming pulmonary engorgement, cessation of flow from the right heart, and death would rapidly ensue. The alternative supposition of the right ventricle persistently discharging into the lung-capillaries an amount of blood actually less than the quantity as persistently set forth by the left ventricle into the systemic circulation, involves a physical contradiction unnecessary to refute. Whatever, therefore, may be the actual capacities of the ventricles, or the quantities which under pressure they may be made to contain, this law must be always paramount to enable the healthy heart to act freely and without the production of a congested or overloaded condition of the pulmonic or systemic circulations; *the quantities of blood entering the ventricles synchronously must be equal*, and the quantities leaving them synchronously must also be equal; and to prevent the occurrence or production of cardiac congestion the quantity of blood received by the ventricles in diastole must equal the quantity expelled by the ventricles in systole, small deviations being allowed within certain limits of health. We shall see the bearing of these latter remarks when we consider the mode in which hearts much diseased in their orifices and valvular apparatus are often enabled to carry on a tolerably unembarrassed circulation, and with but little functional disturbance experienced by the individual so circumstanced.

The anatomy of the organ fully corroborates the principle we are seeking to establish; for we are told that “the capacities of the ventricles are probably equal” (Cruveilhier); and again, “there are reasons for believing that during life any difference between the capacities of the ventricles is very trifling, if it exist at all”†.

And lastly, “the whole, or very nearly the whole of the blood contained in the ventricles is discharged from them at each systole; for the left ventricle is frequently found quite empty after death; and if a transverse section be made through the heart in a state of well-marked rigor mortis (which may be considered as representing its ordinary state of complete contraction), the ventricular cavity is found to be completely obliterated.”

From these considerations we may, I believe, fairly assume that

- (1) { Equal times of ventricular contraction,
 { Equal times of ventricular dilatation,

* Phil. Trans. part 3, 1864.

† Quain's 'Anatomy,' by Dr. Sharpey, vol. iii, p. 255.

- (2) { Equal or almost equal volumes of blood received in diastole,
 { Equal or almost equal volumes of blood expelled in systole,
 (3) Equal or almost equal capacities of ventricles,

are the main characteristics of a heart which is normal in structure and perfect in function.

(1) In employing the words equal times with reference to the periods respectively occupied by the contraction and dilatation of the ventricles, I would wish to refer for a moment to the statements made by our leading authorities as to the average duration of the systole and diastole of the healthy heart.

Dr. Carpenter states that the ventricular contraction occupies $\frac{2}{5}$ and the ventricular dilatation $\frac{3}{5}$ of the time which elapses between two consecutive beats of the pulse. Dr. Walshe informs us that the time from the commencement of the first to the beginning of the second sound is, on an average, one half of the time from pulse to pulse. Dr. Burdon Sanderson, in his *Hand-book of the Sphygmograph*, says, "There are several facts not difficult of observation which show that the time occupied by the heart in contracting is very much shorter than is commonly supposed. The first sound being synchronous with the commencement of the contraction of the ventricles and the closure of the mitral valve, and the second with the closure of the aortic valves, it is clear that the interval between these two events expresses the duration of the contraction of the heart. Now the most unpractised auscultator can readily satisfy himself, while listening to the sounds of a heart contracting sixty times in a minute, that the time between the first and second sounds is not equal to that which separates the second from the first; and that it cannot be admitted for a moment (as stated in our leading physiological text books) that a heart occupies half of a second in contracting."

This statement is borne out in the last edition of Kirkes's '*Physiology*,' edited by Morratt Baker, in which the periods of ventricular contraction and dilatation are considered to be in the ratio of 4 to 7. Chauveau's experiments on the living horse and the sphygmographic tracings of the radial pulse in man, clearly indicate that the times of ventricular contraction and dilatation are very different in duration; and the inferences which are deducible from the study of the comparative areas of the four orifices will fully substantiate the statement that the systole of the ventricles "is a much shorter proceeding than is usually supposed."

(2) And again, with regard to the words "equal volumes of blood" used above, I need scarcely remark that the same volume (quantity, ounces, cubic inches) of blood is not persistently and at all times received by and thrown out of the heart at every complete revolution of the organ. The reverse is, in fact, nearer the truth; for the ventricles (though of course always full from the impossibility of a vacuum existing in their interior) vary considerably from time to time in their degree of fulness and expansion. In profound sleep, or in the perfect rest and muscular relaxation of

the recumbent posture, the flow of blood through the heart is entirely and solely under the control of the heart itself (some allowance being made for the effects of the respiratory movements which "act on the whole advantageously to the circulation"), the right being filled by the contractile energy of the left side of the organ. In our waking moments, however, during exertion, every movement of the body tends to force the blood in the veins in an onward course towards the right chambers of the heart, which would become gorged from over-distension did not the healthy right ventricle assume corresponding energy and force and expel the blood with increased rapidity into the capillaries of the lungs. An increase in the number and depth of the respiratory movements ensues, accelerating the passage of the blood through the lungs to the left side of the heart, which, by an instinctively increased reaction upon its contents, propels the blood forcibly into the systemic circulation. The so-called vital capillary force or interaction between blood and tissue may assist in forwarding the current, but its amount is evidently excessively small in comparison with the enormous contractile energy of the two ventricles. Violent and sudden exertion may for a short time disturb the balance between the two hearts (the cavæ and right auricle in one side, and the pulmonary vessels and left auricle in the other side being, for a time, the safety reservoirs or receptacula of the blood waiting to be forwarded); but with bodily rest equilibrium becomes rapidly reestablished, and equal volumes of blood are again poured forth in equal and the same times from the two ventricles of the heart.

Returning from this digression to the immediate subject of this paper, we have to consider the cause of the differences in the areas of the four principal orifices of the heart.

The right and left sides of that organ are, to all intents and purposes, two distinct and perfect hearts, discharging individually their own proper functions, but associated in one common interest by certain bands of muscular fibres and intercommunicating nerve-ganglia. Now if these two hearts had exactly equal tasks to perform and were simply designed to propel the contents of their ventricles to equal distances and with equal velocities, if, in a word, they had been intended to overcome equal obstacles in the pulmonic and systemic circulations respectively, their walls would have been undoubtedly constructed of equal thickness, and the corresponding orifices of the two sides would have been of equal areas, the tricuspid being equal to the mitral and the pulmonic to the aortic aperture. But as the left ventricle has to propel the blood to far greater distances, and to overcome obstacles much greater than those found in the pulmonic circulation, the velocity and force of the stream sent from the left must be evidently greater than the velocity and force of the blood thrown out by the right ventricle. To secure this result, I need scarcely say that the left is rendered considerably thicker and stronger than the right ventricle by the greater development of its walls; *but here* we must bear in mind the *cardinal fact* (the key to the entire question) that, whatever be the velocity

and force of the streams issuing from the two ventricles, the quantities of blood expelled by the synchronous contraction of the two chambers must be exactly the same, or else accumulation in the pulmonic or systemic systems would ensue, and the machine be brought to a standstill.

As, therefore, the two ventricles contracting with *unequal forces* have to expel *equal quantities* of blood in *equal and the same time* to *unequal distances* and to overcome *unequal resistances*, the perfect synchronism of the ventricular contractions can be only obtained by an exact graduation of the areas of the orifices of the aortic and pulmonary artery to the muscular forces respectively impressed upon the contents of the two ventricles in systole, and consequently to the velocities of the streams issuing from those chambers. The area of the aortic must be therefore smaller than the area of the pulmonic, and *in such proportion* that the normal average contents (say, three ounces) of the left ventricle shall occupy exactly the same time in passing through the aortic as is required by the three ounces of the right ventricle in passing through the pulmonic opening. The greater muscular power of the left, as compared with that of the right ventricle, causes a corresponding greater velocity and force of the column of blood issuing from its outlet, while the smaller area of the aortic, as compared with that of the pulmonic opening, exactly equalizes the times occupied by the contractions of the two chambers. Without such an arrangement in the comparative areas of the two outlets, it is clear that the stronger left would completely empty itself before the right ventricle had accomplished the same function, and the synchronous action of the two hearts would be thus rendered impossible. Equal quantities of blood are, however, in the way described, made to pass exactly synchronously through the aortic and pulmonic openings, but with, of course, unequal velocities, the blood-particles which traverse the narrow aortic travelling with greater speed than those which pass through the larger pulmonic orifice. Mathematically expressed, the velocities of the streams through the orifices are inversely as the areas of those orifices, or

$$\frac{\text{velocity through aortic opening}}{\text{velocity through pulmonic opening}} = \frac{\text{area of pulmonic opening}}{\text{area of aortic opening}}$$

And if we assume the mean measurements of the orifices found in the former part of this paper to be correct,

$$\begin{aligned} \left. \begin{array}{l} \text{the velocity through} \\ \text{aortic opening} \end{array} \right\} &= \frac{1 \text{ sq. inch}}{.75 \text{ sq. inch}} \times \text{velocity through pulmonic opening} \\ &= 1.3 \text{ time the velocity through pulmonic opening} \\ &= 1\frac{1}{3} \text{ time the velocity through pulmonic opening,} \end{aligned}$$

or, in other words, the velocities of the currents through the aortic and pulmonic orifices are in the ratio of 4 to 3.

The arguments which I have advanced respecting the aortic and pulmonic will be equally applicable to the tricuspid and mitral openings; for:—

1st. The two ventricles are exactly synchronous in their diastole, re-

ceiving their respective charges of blood from the auricles in exactly equal and the same times.

2nd. Equal volumes of blood enter the two ventricles during their diastole, or else accumulation and stagnation would ensue: I am speaking here of healthy ventricles.

3rd. The ventricles are of equal capacities; but

4th. As the currents which traverse the tricuspid and mitral orifices are of unequal velocities, the areas of those openings must be of such magnitudes that equal volumes of blood must pass through them in exactly equal and the same times. The tricuspid having a slower velocity than the mitral current, will necessitate the area of the tricuspid being proportionally larger than the area of the mitral orifice. In a word, the synchronous dilatation of chambers, admitting equal volumes of blood must entail such a relation of area between the two inlets that

$$\frac{\text{the velocity through tricuspid}}{\text{the velocity through mitral}} = \frac{\text{area of mitral}}{\text{area of tricuspid}}.$$

And if we assume the measurements previously found to be correct,

$$\text{the velocity through tricuspid} = \frac{1.25}{1.75} \text{ velocity through mitral,}$$

$$= \frac{5}{7} \text{ velocity through mitral,}$$

i. e. the velocities of the currents of blood in diastole through the tricuspid and mitral orifices are in the ratio of 5 to 7.

It may be fairly asked what proofs can be given that the velocities of the currents of blood which traverse the tricuspid and mitral orifices are unequal, and that the mitral incoming stream possesses a stronger ventricular dilating power than the current which enters the tricuspid to expand and fill the right ventricle. I shall refer to this point shortly; but whatever may be the value of the reasons which will be adduced in support of the above view, there can be no doubt, *in fact*, that the two orifices in healthy hearts always differ in size, and the synchronous expansion of ventricles with unequal inlets must inevitably lead to this result—that the larger must admit a current of correspondingly smaller velocity than that which traverses the smaller opening; or, mathematically expressed, the velocities of the incoming tricuspid and mitral streams must be inversely as the areas of the orifices.

From the data at which we have arrived, and estimating the mean amount of the ventricular contents at three ounces (or five cubic inches, nearly), although it must be confessed that this is an uncertain estimate, we may readily calculate the average velocities of the currents which traverse the four orifices. We shall consider the pulse to beat at the rate of 70 per minute, and the periods of ventricular contraction and dilatation to be in the ratio of 1 to 2, *i. e.* the ventricular contraction occupying one-third of the time between two pulses; *i. e.*

$$= \frac{1}{3} \text{ of } \frac{1}{70} = \frac{1}{210},$$

1. *Aortic Orifice.*

$$\begin{aligned}
 \text{Velocity through aortic orifice} &= \frac{\text{volume expelled in } \frac{1}{210}'}{\text{area of aortic orifice}} \\
 &= \frac{5 \text{ cub. inches}}{.75 \text{ sq. in.}} \text{ in } \frac{1}{210}' \\
 &= 23.1 \text{ inches in one second} \\
 &= 2310 \text{ yards per hour.}
 \end{aligned}$$

2. *Pulmonic Orifice.*

$$\begin{aligned}
 \text{Velocity through pulmonic orifice} &= \frac{3}{4} \text{ velocity through aortic opening} \\
 &= 17.3 \text{ inches in one second} \\
 &= 1725 \text{ yards per hour.}
 \end{aligned}$$

3. *Tricuspid Orifice.*

$$\begin{aligned}
 \text{Velocity through tricuspid} &= \frac{5 \text{ cub. inches}}{1.75 \text{ sq. in.}} \\
 &= \frac{1}{.35} \text{ inch in } \frac{1}{105} \text{ l'} \\
 &= 5 \text{ inches in one second} \\
 &= 500 \text{ yards per hour.}
 \end{aligned}$$

4. *Mitral Orifice.*

$$\begin{aligned}
 \text{Velocity through mitral} &= \frac{7}{5} \text{ velocity through tricuspid} \\
 &= 7 \text{ inches in one second} \\
 &= 700 \text{ yards per hour.}
 \end{aligned}$$

The mean velocities of the currents of blood traversing a healthy heart, with the dimensions of the areas as given above, are as follows:—

	yards.	mile.
Aortic	= 2310	= 1.3 per hour
Pulmonic	= 1725	= 1 nearly
Mitral	= 700	= .4
Tricuspid	= 500	= .28

In such a heart we see, therefore, that the blood enters the tricuspid orifice at the rate of nearly $\frac{1}{4}$ mile per hour, and leaves it through the aortic orifice at the rate of nearly $1\frac{1}{4}$ mile per hour; and that the velocity, therefore, of the tricuspid incoming current is only one-fifth of the velocity of the stream which passes through the aortic orifice.

Without entering into arithmetical details, such a result as the above is easily arrived at when we bear in mind the facts that the same quantity of blood passes through the two openings, but that while the tricuspid is, according to Dr. Peacock, $2\frac{1}{2}$, and according to Dr. Reid nearly three times larger than the aortic orifice, the flow of the three ounces through the former occupies nearly twice the time required by

the passage of the same quantity of blood through the latter opening. The tricuspid is nearly three times larger than the aortic aperture, and is open for the transmission of the same volume of blood more than double the length of time occupied by the latter opening. Hence the comparative slowness of the incoming tricuspid current.

These speculations upon the absolute and relative velocities of the currents of blood through the heart are not without practical value, inasmuch as they have a direct bearing upon the question of the amount of pressure exerted by that fluid in each chamber of the organ, and are links in the chain of reasoning respecting the comparative areas of the four orifices. The first-recorded experiments to determine this pressure were made by Dr. Stephen Hales, F.R.S., and were published by him in his 'Statical Essays' in 1732. Thus, when tubes were fixed into the crural artery and jugular vein of different animals, the heights to which the blood rose were found to be as follows :—

	Artery.	Vein.
Horse	114 inches.	12 inches.
Sheep	77½	5½
Dog	48	4½

These experiments were, of course, rather roughly made and without modern appliances; but they serve to show that the pressure of the blood in the jugular vein is only one-ninth to one-fourteenth of the pressure observed in the arterial side of the circulation. Valentin, by means of the hæmadynamometer, estimated the pressure in the jugular vein to be one-tenth to one-twelfth of the pressure in the carotid artery, and "in the upper part of the inferior vena cava could scarcely detect the existence of any pressure, nearly the whole force from the heart having been apparently consumed during the passage of the blood through the capillaries" (Kirkes and Paget).

It is thus sufficiently clear, experimentally, that the velocity and momentum of the blood which enters the right auricle and finds its way into the right ventricle must be very small in comparison with the rapidity and momentum of the current issuing from the left ventricle; and we can therefore, from this fact, understand that the tricuspid is constructed of much greater area than the aortic opening, in order that its much larger orifice may compensate for the comparatively sluggish stream which it has to transmit. It is evident enough why the blood which has returned to the right heart possesses so small an amount of velocity and momentum. In its passage through the systemic circulation it has encountered and overcome an amount of obstruction which, by the time it has arrived in the right auricle, has deprived it of the greater portion of the velocity and momentum which it had derived from the contractile energy of the left ventricle, assisted, as that power has been, by the muscular pressure on the veins of the body. The columns of blood from the superior and infe-

rior venæ cavæ enter the auricle, therefore, slowly, and with small force, but with an amount of velocity and momentum exactly adapted to and sufficient for the expansion of the right ventricle. It cannot be for one moment maintained that the right is weaker than the left heart in proportion to the work to be done by the respective sides, for each organ is exactly adapted to the task which it has to perform, and the perfection of the mechanism is as manifest in one as in the other side of the heart. The right side is undoubtedly exposed to sudden and great variations in the amount of blood-pressure to which it is from time to time subjected; but there can be no reason for believing that provision has not been made for such variations *within due limits*. In fact daily experience shows us how the right side will maintain its vigour unimpaired, although severely and often tried by the alterations in the blood-pressure resulting from rapid walking, running, pulling at the oar, and the usual athletic exercises. The slowness of the current which returns to the right side, the large area of the tricuspid orifice, and the comparatively long period of time during which the ventricle is open to receive its contents, evidently confirm the view that the right ventricle offers but little resistance to the incoming current, and that a stream of small velocity and energy is amply sufficient to fill and complete the expansion of that chamber. The force which is exerted by the contraction of the auricle is small, and in operation for a short period of time ($\frac{1}{8}$ to $\frac{1}{10}$ of a minute), and is chiefly of use, I believe, in completing the closure of the tricuspid valve in the manner described by Baumgärtner, Valentin, and Halford. It must be also borne in mind that the particles of the blood-stream which have entered the tricuspid orifice in a direction nearly at right angles to the axis of the pulmonary artery must, when the ventricle has become filled, change their direction of motion to find their way in systole out of the ventricle. At the end, therefore, of the diastole I imagine that the whole of the contents of the ventricle is at rest (momentarily, but not less really so), and ready to take up a new movement in a course nearly at right angles to its line of entrance from the auricle. If this view, which has escaped the attention of physiologists, be correct, we observe an additional reason for the blood which enters the ventricle possessing an amount of velocity and energy just sufficient, and no more, to complete the dilatation of the chamber, and having performed its task, to assume for a moment an attitude of repose before the contraction of the ventricle sends it forth in a different direction. All force in the human body is economized, the means is strictly adapted to the end: the left ventricle puts forth power sufficient to carry the blood through the systemic capillaries to the right side of the heart; the blood enters the right auricle with an amount of pressure sufficient, with the aid of the auricular contraction, to fill the ventricle, and should any excess of momentum exist, it is probably annihilated by the incoming current meeting the dense interlacement of the fibres of the columnæ carneæ which form such prominent parts of the interior of both ventricles. It is

interesting to observe that this interlacement is most dense near the apex, where the incoming current impinges with greatest force, and where the excess of momentum of the blood can be easier annihilated before it changes its direction of motion to escape in systole from the chamber. The remarks which belong to the right are equally applicable to the left ventricle, and lead to the conclusion that the current entering through the mitral orifice, as soon as the chamber is filled, loses its motion for a moment before the contractile force of the ventricle launches it forth in a totally different direction through the aortic opening into the systemic circulation.

I will now return to the statement which I had left unproved, that the velocities of the synchronous tricuspid and mitral currents are unequal, and that the latter possesses a stronger ventricle-dilating power than the former.

The argument to establish this point is very brief.

The two ventricles being of unequal thickness and containing consequently unequal quantities and weights of muscular fibre, will necessarily require currents of blood of unequal momenta to overcome their respective *inertia*, fill their chambers, and complete their dilatation in exactly equal and the same times. That is,

the momentum of the mitral is greater than the momentum of the tricuspid current ;

or, in other words,

the volume of the mitral column multiplied by its velocity *is greater than* the volume of the tricuspid column multiplied by its velocity ;

but the volume of each current *is the same* ; hence, eliminating *volume* from each side of the above, it is evident that

the velocity of the mitral is greater than the velocity of the tricuspid current, the conclusion which was to be demonstrated*.

In concluding this paper, I would very briefly recapitulate the conclusions at which I have arrived. I have proved, from the measurements of the orifices made by Drs. Peacock and Reid, that the areas of the openings in man are subject to a constant law, summarily expressed thus :—

$$\frac{T}{M} = \frac{P}{A}.$$

And the same result I have also obtained from my own measurements of

* I have employed the word momentum as one in common use ; but the term *vis viva* would have been more correct, inasmuch as the latter expression represents the mass of a body in motion multiplied by the *square* of its velocity. In this case, therefore, while the velocities of the tricuspid and mitral moving masses of blood are in the ratio of 5 to 7, the energy or *vis viva* of the tricuspid is to the energy or *vis viva* of the mitral current or mass as 25 to 49. This observation does not affect the reasoning employed above, but shows the greatness of the disparity between the ventricle-dilating powers of the two incoming currents.

the healthy heart. Furthermore, I have proved, from my own measurements, that the same law probably regulates the areas of the orifices in animals generally; and I have cited several examples in corroboration of the statement. From the existence of this law, it is clear that if the areas of any three healthy orifices be known, the area of the fourth can be determined by calculation.

I have then drawn attention to the curious and important fact which appears to be almost general in animals, that

$$\frac{T}{M} = 1.3 \text{ to } 1.4, \text{ nearly;}$$

and

$$\frac{P}{A} = 1.3 \text{ to } 1.4, \text{ nearly;}$$

consequently that the dimensions of the openings of one side of the heart being given, the areas of the corresponding orifices on the other side of the organ may be obtained by arithmetical process.

Having shown from measurements that the orifices arranged in the order of their magnitude are as follows,

1. Tricuspid,
2. Mitral,
3. Pulmonic,
4. Aortic,

I have sought to determine the reasons for this arrangement, to which however, I shall not again refer.

I propose on some future occasion to show how widely this "*law of the orifices*" which I have discovered is applicable to the heart in its diseased state, and how it serves to explain many important and interesting points relative to the organ. I shall conclude with citing a few instances in which its application throws some light on the effects of pulmonary disease upon the areas of the orifices.

1. *Phthisis* (Dr. Peacock).

	Circumference.	Area.
Tricuspid	51 lines.	207 square lines.
Pulmonic	39	121
Mitral	18	183
Aortic	36	103

$$\frac{T}{M} = \frac{207}{183} = 1.13$$

$$\frac{P}{A} = \frac{121}{103} = 1.17$$

$$\text{Difference of the ratios} = .04$$

2. *Phthisis* (Dr. Peacock).

Tricuspid	43 lines.	147 square lines.
Pulmonic	29	67
Mitral	37	109
Aortic	26	54

$$\frac{T}{M} = \frac{147}{109} = 1.35$$

$$\frac{P}{A} = \frac{67}{54} = 1.25$$

Difference of the ratios = $\cdot 10$

In these two cases the orifices evidently closely exhibit the usual normal relation to each other; and as the blood in phthisis emaciates and diminishes in quantity like the other parts of the body, we should not expect in a pure case of phthisis an amount of pulmonary obstruction sufficient to produce marked alterations in the areas of the openings.

3. *Bronchitis* (Dr. Peacock).

Tricuspid	60 lines.	286 square lines.
Pulmonic	45	193.07
Mitral	54	232.1
Aortic	26	103.18

$$\frac{T}{M} = \frac{286.6}{232.1} = 1.235$$

$$\frac{P}{A} = \frac{193.07}{103.18} = 1.870$$

Difference of the ratios = $\cdot 635$

The relation is clearly abnormal.

T and M are nearly in normal relation to each other, but P is abnormally large in relation to A. This heart was enlarged (weighed 14 ounces) from hypertrophy and dilatation of its right side, and the pulmonic orifice, from the abnormal increase of the pressure of the current of blood sent through it by the thickened right ventricle, became considerably dilated.

4. *Bronchitis* (Dr. Davies).

Tricuspid	5 in.	2 sq. in.
Pulmonic	3.5	.975
Mitral	3.8	1.15
Aortic	3.1	.79

$$\frac{T}{M} = \frac{2}{1.15} = 1.74$$

$$\frac{P}{A} = \frac{.975}{.79} = 1.24$$

Difference of the ratios = $\cdot 50$

The tricuspid is evidently abnormally large in relation to the mitral orifice.

If we suppose the other three orifices to be nearly normal (and it is evident that the pulmonic and aortic bear their normal relation to one another), we can calculate the excess of dilatation exhibited by the tricuspid opening

$$\frac{T}{M} = \frac{P}{A};$$

$$\therefore T = 1.15 \times \frac{.975}{.79} = 1.42 \text{ sq. inch.}$$

Hence the tricuspid is .58 inch in excess of its normal area, or more than one-third of its proper size in excess.

5. *Bronchitis* (Dr. Peacock).

The heart in this case weighed eleven ounces. As this does not much exceed its usual weight, it is clear that the pulmonic obstruction was slight. We should therefore expect to find but little deviation from the "law of the orifices."

Tricuspid	62 lines.	306 sq. lines.
Pulmonic, ...	45	161
Mitral	54	232
Aortic	39	121

$$\frac{T}{M} = \frac{306}{232} = 1.32$$

$$\frac{P}{A} = \frac{161}{121} = 1.33$$

$$\text{Difference of the ratios} = .01$$

This result fully bears out the inferences above made.

II. "On the Estimation of Ammonia in Atmospheric Air." By HORACE T. BROWN, Esq. Communicated by Dr. FRANKLAND. Received February 19, 1870.

In the attempts that have been hitherto made to estimate the ammonia present in atmospheric air, the results arrived at by the various experimenters have differed so widely that it is still a matter of uncertainty what the quantity really is. That it is a very small amount all agree, but the extreme results on record vary as much as from 13.5 to .01 part of carbonate of ammonium per 100,000 of air. It may therefore not be without interest to give an account of a simple method affording very concordant and, I believe, accurate results, at the same time being easy of performance and requiring but little time for an experiment.

The apparatus used consists of two glass tubes, each of about 1 metre